Technical Report:

Continuous Gathering with Finite Visibility and Bearing Only – Simulator and Theorem

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1. Introduction

In this report we summarized our final project for the course Advanced Topics in Computer Science - On Multi-A(ge)nt Systems by Professor Alfred M. Bruckstein , Computer science department, Technion – Israel institute of technology. Our project dealt with the problem of Continuous Gathering with Finite Visibility and Bearing Only. As part of the project we created a simulator described in section 2, as well as a brief of the theoretical background of the problem (section 3).

2. Simulator User Manual

In this section, we present the different features implemented as part of the Continuous Gathering with Finite Visibility and Bearing Only Simulator. The simulator was implemented using Net Logo software (version 6.1.1).

After activating the simulator, the following screen should be presented.

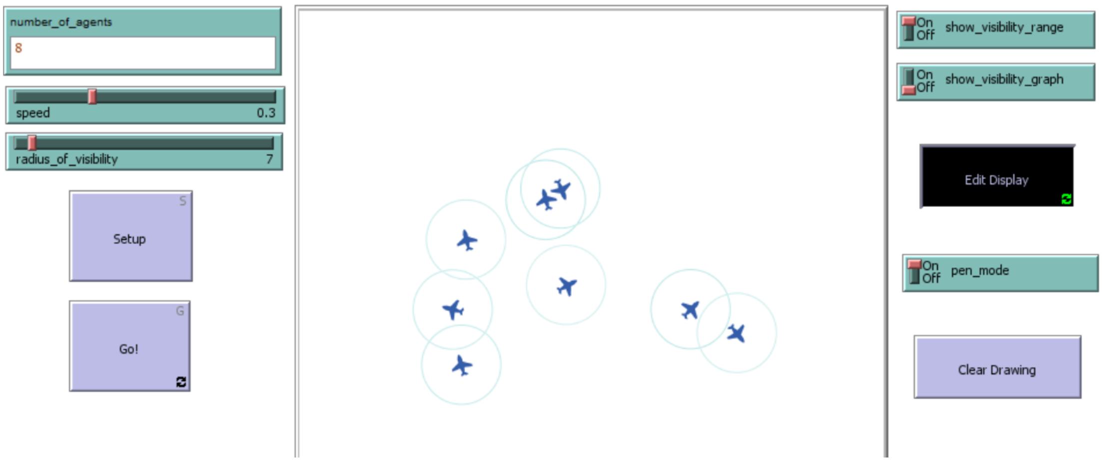
The opening screen allows user to setup a gathering scenario and then execute the gathering algorithm described in the next section (2). We shall brief each button in the simulator.

Setup – The setup button initializes and display a gathering scenario based on the selected features.

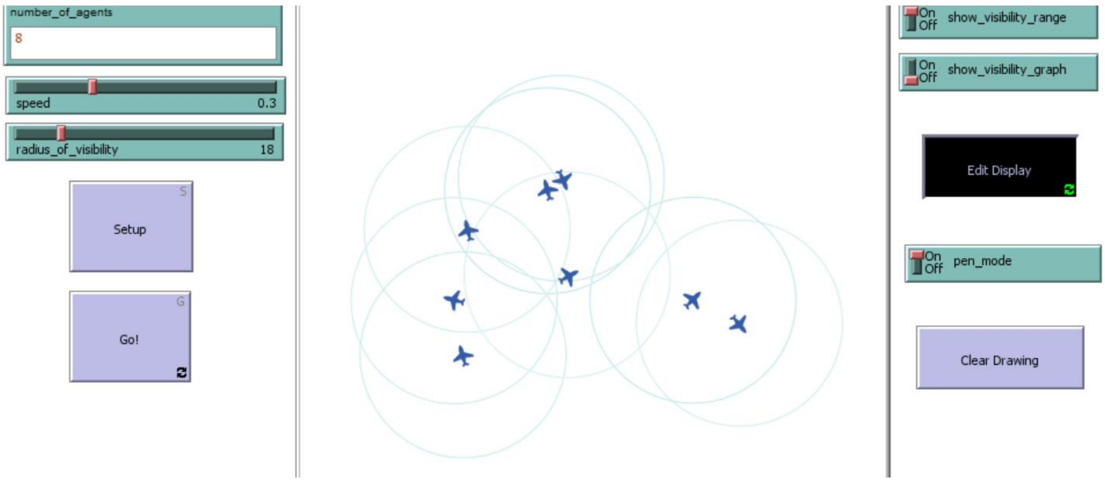
Number of Agents – The number of agent text box allow user to select the amount of agents in the gathering problem.

Edit Display – The Edit Display button allows user to change the displayed features on screen (in addition to the agents). It is recommended to push this button and use the following features in order to build interesting scenarios.

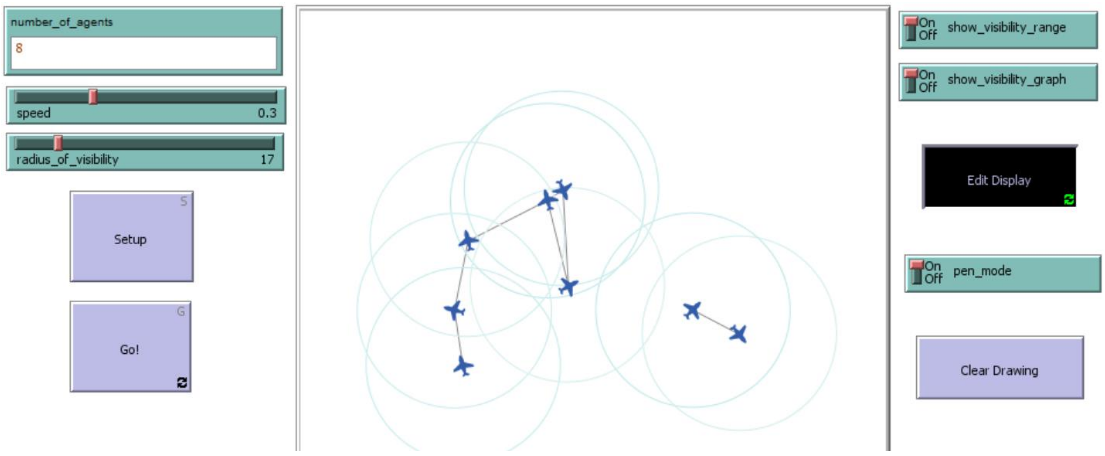
Show Visibility Range - The show Visibility Range button allows user to see each agent visibility range (V).



Radius of Visibility – The Radius Of Visibility slider enable user to change the visibility radius of the agents

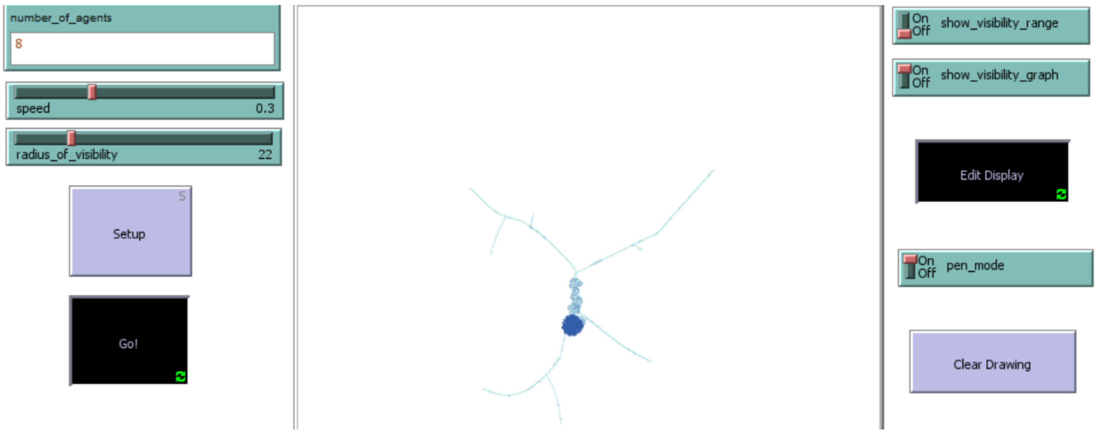


Show Visibility Graph - The show Visibility Graph button Display the visibility graph, enable the user to check the graph is connected. In the following figure, the graph not connected.

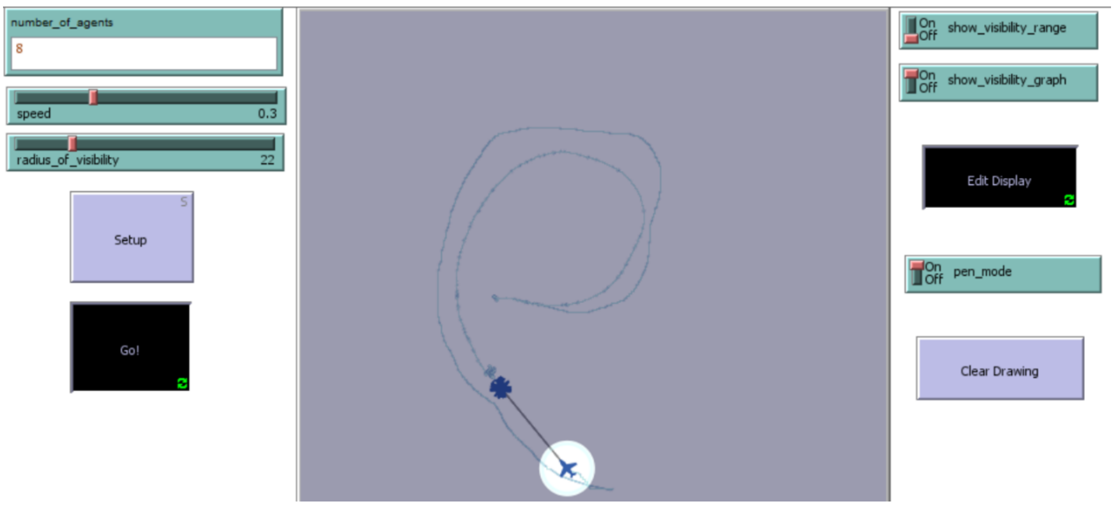


Pen Mode – the Pen mode button display each agent trajectory on screen

Go! – The Go button initiate the gathering process. In the next figure you can see the successful finish of such process using the pen mode.



Drag n' Drop – The simulator allows the user to drag and drop agent during the gathering process (after Go!) or during the setup process (after pushing Edit Display). In the following figure, you can see how we took a single agent after gathering, drag it around the screen and make the other agent chase it. Interesting to see the chain pursuit proof concept – the agents route (inner line) is shorter than the dragged agents route (outer line).



3. Theoretical Analysis

In this section we bring some theoretical explanation for the agents' behavior in our simulator. This analysis is a brief version of full analysis described in [1].

We assume that every agent has information only about the relative bearing direction to its neighbors within a visibility range V (but cannot measure relative distances). First, we present the gathering process based on bearing-only sensing with limited visibility in the continuous time framework (section 3.1). Next, we present a theoretical proof for gathering under connected neighborhood graph (section 3.2).

3.1. Model Dynamics

Assume each agent moves with a constant speed σ unless it stops, according to the following dynamic law:

$$\dot{p}_i(t) = \begin{cases} \sigma\ \hat{p}_{i_{bisector}}(t), & \psi_i(t) < \pi \\ 0, & o.w. \end{cases}$$

Where:

$$\hat{p}_{i_{bisector}}(t) = \frac{\hat{p}_{i_R}(t) + \hat{p}_{i_L}(t)}{\|\hat{p}_{i_R}(t) + \hat{p}_{i_L}(t)\|}$$

the points $p_{iR}(t)\ and\ p_{iL}(t)$ being the positions of the extreme right and left agents defining the minimal angular sector containing all neighbors of agent i. This sector angle will be denoted by $\psi i\ = \angle p_{iR}\, p_i\, p_{iL}$.

$\hat{p}_{i_R}(t)$ and $\hat{p}_{i_L}(t)$ are unit vectors so that:

$$\hat{p}_{i_R}(t) = \frac{p_{i_R}(t) - p_i(t)}{\|p_{i_R}(t) - p_i(t)\|}$$

and

$$\hat{p}_{i_L}(t) = \frac{p_{i_L}(t) - p_i(t)}{\|p_{i_L}(t) - p_i(t)\|}$$

Informally, agent i moves at a constant velocity σ along the bisector of $\psi_i(t)$, unless $\psi_i(t)\ \geq\ \pi$ then agent i doesn't move (see the following figure 21 from [1]).

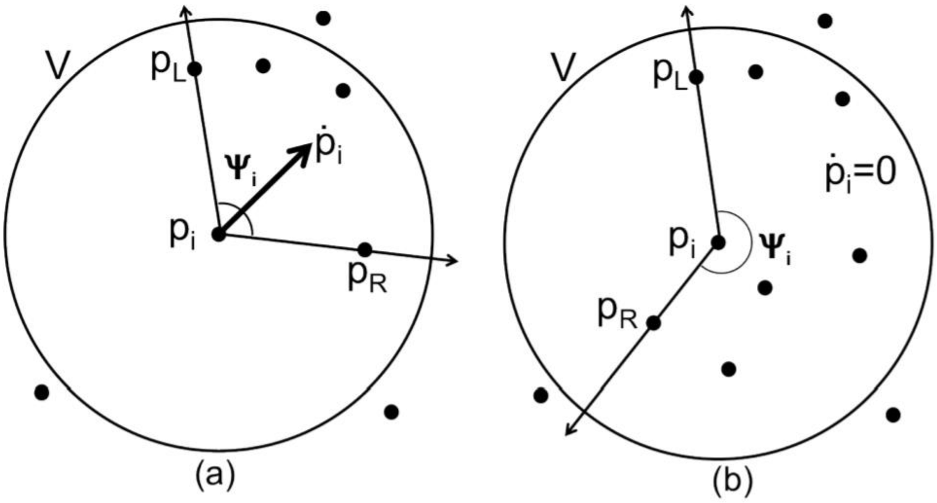


Figure 21: (a) Agent i moves in the direction of ψ_i's bisector. (b) Agent i does not move since $\psi_i > \pi$

3.2. Gathering proof

In this section we give a summarize proof of the following theorem (theorem 8 in [1]):

Theorem 8: For an initial constellation of a connected neighborhood graph, all the agents of system S7 converge to a point in finite time.

Proof. We shall prove Theorem 8 using three steps:

Step 1: Any two neighbors of system S7 will remain neighbors (Lemmas 15 and 16).

Step 2: At any time t the convex-hull of system S7 is contained in its previous (Lemma 17).

Step 3: While the convex-hull of the agents locations has a perimeter greater than zero, the perimeter decreases at a finite speed (Lemma 18).

These three lemmas prove that system S7 converges to a point in finite time as claimed.

3.2.1. Step 1: Any two neighbors of system S7 will remain neighbors

We shall give some informal intuition for lemmas 15 and 16 proved in [1].

Let us first define an allowable region ARi(t) where each agent i can move without losing any existing neighbor, i.e. its distance from every existing neighbor will stay smaller than V .

The current allowable region, where agent i may move without losing any of its existing neighbors, is denoted by (see the following figure 22 from [1]):

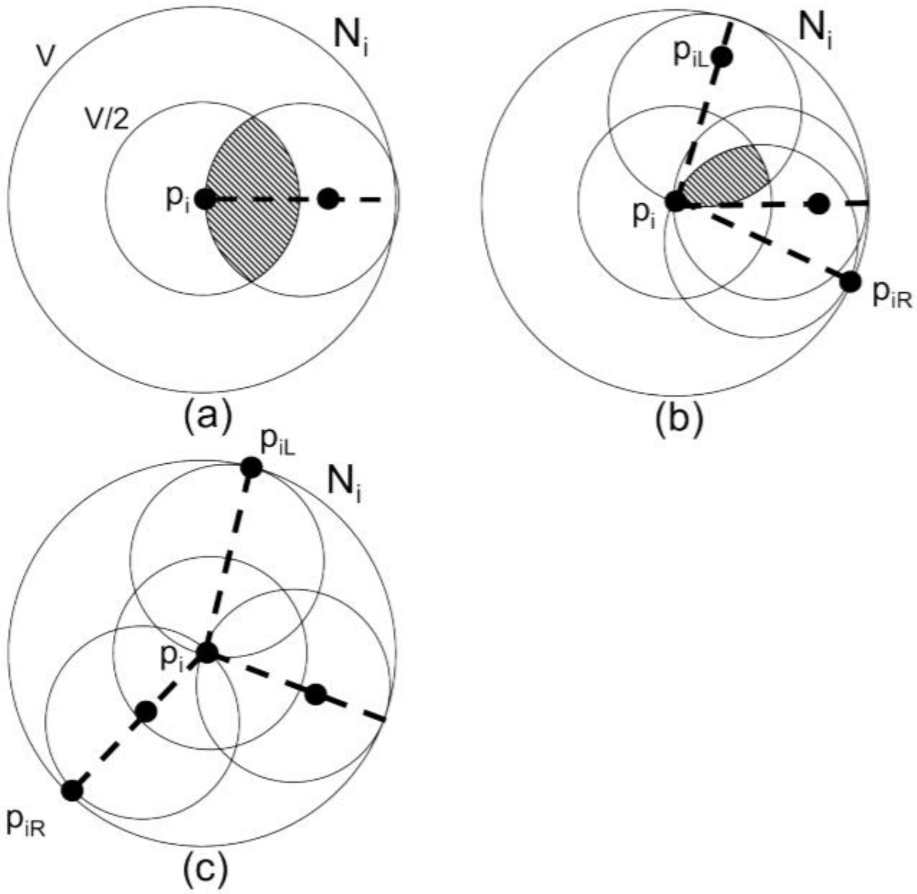


Figure 22: Allowable regions for agent i. (a) Single neighbour. (b) Intersection between the extreme left agent's disc, the extreme right agent's disc, and the disc $D_{\frac{V}{2}}(p_i)$. (c) No allowable region since the intersection yields an empty area.

Lemma 15. If all agents move to an arbitrary location inside their allowable regions they will not lose any of their neighbors.

Proof. See [1]

Lemma 16. Motion according to the dynamics rule ensures that all agents move into their allowable regions.

Proof. See [1],

Intuitively by moving according to the described dynamic, agent takes a step only if it gets him closer to all its neighbors. Therefore, it is impossible to loose the visibility graph connectivity.

3.2.2. Step 2: At any time t the convex-hull of system S7 is contained in its previous

Lemma 17: Let CH(P(t)) be the convex-hull of agents' positions in system S7 at time t. Then for all t ≥ 0 and ∆t > 0

$$CH(P(t + \Delta t)) \subseteq CH(P(t))$$

Proof. By the dynamic law each agent moves along the bisector of $\psi_i(t)$, $where\ \psi_i(t)$ is the angle of the smallest wedge containing all the neighbors of agent i. And since there is no agent located outside the convex-hull of the system, no agent moves out of the convex-hull.

3.2.3. Step 3: While the convex-hull of the agents locations has a perimeter greater than zero, the perimeter decreases at a finite speed

Lemma 18. If the graph topology of system S7 is connected and the perimeter of its convex-hull is greater than zero, then the perimeter of its convex-hull decreases at a rate bounded away from zero.

Proof. We shall show that the perimeter of CH(P(t)) drops at a strictly positive rate as long as the diameter of the system is strictly positive.

The proof is based on the dynamics of the agent (or agents) s, located at a current sharpest corner of the system's convex-hull. Let ϕ_s be the inner angle of this corner.

The sum of angles of any convex polygon is $\pi(m-2)$, where m is the number of its corners, therefore the angle of its sharpest corner $\phi_s$is at most $\pi(1 - 2\ m\ )$. System S7 contains n agents, hence the system's convex-hull has m ≤ n corners. We denote the upper limit on the sharpest corner of the convex-hull by ϕ_*, so

$$\varphi_s \leq \varphi_* = \pi(1 - 2/n)$$

Define L(P(t)) as the perimeter of CH(P(t)) and $l_i(t)$ as the length of the convex-hull side connecting corners i and i + 1 at time t.

Let $\phi_i(t)$ be the angle of corner i of CH(P(t)), let $\alpha_i(t)$ denote the direction of motion of the agent located at corner i relative to the direction of corner i+1, and let $v_i(t)$ be the speed of the agent located at corner i (as shown in Figure 25).

We have that

$$\lim_{dt\to 0^+}\{l_i(t+dt)-l_i(t)\} =$$

$$-(v_i(t)\cos\alpha_i(t)+v_{i+1}(t)\cos(\varphi_{i+1}(t)-\alpha_{i+1}(t)))dt+\mathcal{O}(dt)$$

hence,

$$\dot{\mathcal{L}}(P(t)) = -\sum_{i=1}^{m} v_i(t)(\cos\alpha_i(t)+\cos(\varphi_i(t)-\alpha_i(t)))$$

$$= -\sum_{i=1}^{m} v_i(t)\cos(\frac{\varphi_i(t)}{2})\cos(\frac{\varphi_i(t)-2\alpha_i(t)}{2})$$

Let $v_s(t), \phi_s(t)$ *and* $\alpha_s(t)$ be the relevant values associated with agent s. Since $v_s = \| \dot{p}_s(t) \|$ is positive and bounded away from zero by the assumed rule of motion (47), and by Lemma 18 we have $\alpha_i(t) \le \phi_i(t) < \pi$ we have

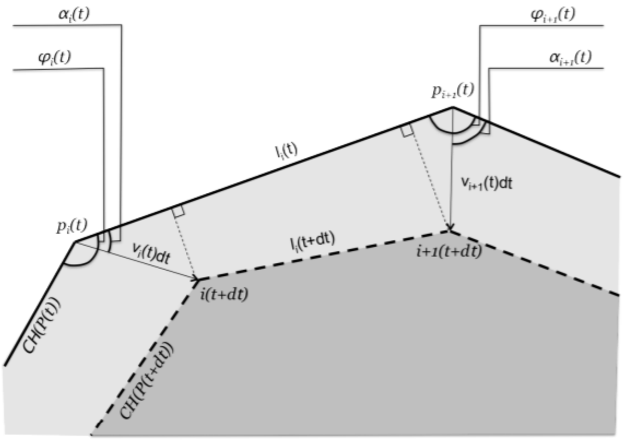


Figure 25: Convex-hull shrinkage.

Hence, the perimeter decreases at a rate of at least $\sigma\cos^2(\phi_*/2)$ proving Lemma 18

To prove Theorem 8 we have by Lemma (18) that the length of the perimeter of the convex-hull of system S7 decreases at a bounded away form zero rate, therefore it necessarily converges to a point in finite time as claimed.

References

[1] - Barel A., Manor R., and Bruckstein A.M ,called "COME TOGETHER: MULTI-AGENT GEOMETRIC CONSENSUS (GATHERING, RENDEZVOUS, CLUSTERING, AGGREGATION)"